



Analysis 3 - Exercise Sheet 8

Publication date: November 30, 2022

Due date: Dezember 7, 2022

Exercise 8.1 (20 pts) Consider the set of unitary matrices $O(n) = \{A \in \mathbb{R}^{n \times n} \mid A^{-1} = A^T\} \subset \mathbb{R}^{n \times n}$.

- Show that $O(n)$ is connected.
- Show that the tangent space of $O(n)$ at the identity matrix is the set of skew-symmetric matrices.

Exercise 8.2 (20 pts) Show that a topological space with a connected, dense subset is connected.

Exercise 8.3 (20 pts) Let A, B be subspaces of a topological space such that $A \cup B$ and $A \cap B$ are connected. Prove that if A, B are both closed or both open, A and B are connected.

Exercise 8.4 (20 pts) Determine the tangent space of the surface defined by $x^2 + y^2 - z^2 = 25$ in all points where $z = 0$.

Exercise 8.5 (20 pts) Where does the mapping $\mathbb{R}^2 \rightarrow \mathbb{R}^3, (u, v) \mapsto (x, y, z)$ via

$$x = u \cos(v), \quad y = u \sin(v), \quad z = u^2 + v^2$$

locally define a regular 2 dimensional hyperplane? Compute the tangent space in $(1, 0, 1) \in \mathbb{R}^3$.