POLYNOMIAL DIVISION

EXAMPLE

We want to divide

$$x^2 + 10x + 21$$ DIVIDEND

by

$$x + 7$$ DIVISOR

STEP 1 Divide 1st term of DIVIDEND with 1st term of Divisor

Write Result as first term of QUOTIENT

In this case

$$\frac{x^2}{x} = x$$

$$x + 7 \;\overline{)\; x^2 + 10x + 21} \qquad \begin{matrix} x \\ \text{QUOTIENT} \end{matrix} \qquad \text{DIVIDEND}$$

DIVISOR

STEP 2 Multiply QUOTIENT By DIVISOR

WRITE Result bELow DIVIDEND

$$\begin{array}{r|l} & x \\ \hline x+7 & x^2+10x+21 \\ & x^2+7x \end{array}$$

x — QUOTIENT

$x^2+10x+21$ — DIVIDEND

$x+7$ — DIVISOR

Step 3 Subtract the Result from DIVIDEND

OUTCOME IS NEW DIVIDEND

$$\begin{array}{r|l} & x \\ \hline x+7 & x^2+10x+21 \\ & x^2+7x \\ \hline & 0+3x+21 \end{array}$$

x — QUOTIENT

$x^2+10x+21$ — DIVIDEND

$x+7$ — DIVISOR

$0+3x+21$ — NEW DIVIDEND

STEP 4 DIVIDE FIRST terme of new DIVIDEND by first Terme of DIVISOR

Write result as 2nd terme of QUOTIENT

In this case we have

$$\frac{3x}{x} = 3$$

$$\begin{array}{r} x+3 \quad \text{QUOTIENT} \\ x+7 \;\overline{)\; x^2+10x+21} \quad \text{DIVIDEND} \\ \text{DIVISOR} \quad x^2+7x \\ \hline 0+3x+21 \quad \text{NEW DIVIDEND} \end{array}$$

STEP 5 Multiply 2nd TERM of QUOTIENT By Divisor

WRITE Result below NEW DIVISOR

$$
\begin{array}{r|l}
 & x+3 \quad \text{QUOTIENT} \\
x+7 & x^2+10x+21 \quad \text{DIVIDEND} \\
\text{Divisor} & x^2+7x \\
\hline
 & 0+3x+21 \quad \text{NEW DIVIDEND} \\
 & \quad 3x+21
\end{array}
$$

STEP 6 Subtract the Result from NEW DIVIDEND

$$
\begin{array}{r|l}
 & x+3 \quad \text{QUOTIENT} \\
x+7 & x^2+10x+21 \quad \text{DIVIDEND} \\
\text{Divisor} & x^2+7x \\
\hline
 & 0+3x+21 \quad \text{NEW DIVIDEND} \\
 & \quad 3x+21 \\
\hline
 & 0 \quad 0 \quad 0 \quad \text{REMAINDER}
\end{array}
$$

Step 7 AS NEW DIVIDEND IS ZERO, the Algorithm Stops. Result is

$$\text{DIVIDEND} = (\text{DIVISOR} \times \text{QUOTIENT}) + \text{REMAINDER}$$

In our example

$$x^2 + 10x + 21 = (x + 7)(x + 3)$$

SUMMARY OF ALGORITHM

1. Divide 1st term of DIVIDEND with 1st term of DIVISOR

   WRITE Result as first term of QUOTIENT

2. Multiply QUOTIENT BY DIVISOR

   WRITE Result below DIVIDEND

3. Subtract the Result from DIVIDEND

OBTAIN NEW DIVIDEND

4. REPEAT UNTIL NEW DIVIDEND HAS LOWER DEGREE OF DIVISOR

5. STOP. RESULT IS

$$\text{DIVIDEND} = (\text{DIVISOR} \times \text{QUOTIENT}) + \text{REMAINDER}$$

EXAMPLE DIVIDE $6x^3 + 5x^2 - 7$ (DIVIDEND) by $3x^2 - 2x - 1$ (DIVISOR).

$$3x^2 - 2x - 1 \;\big|\; 6x^3 + 5x^2 - 7$$

$2x + 3$ — QUOTIENT

DIVISOR: $3x^2 - 2x - 1$ | DIVIDEND: $6x^3 + 5x^2 - 7$

$6x^3 - 4x^2 - 2x$

$9x^2 + 2x - 7$

$9x^2 - 6x - 3$

$8x - 4$ — REMAINDER

The outcome is

$$6x^3 + 5x^2 - 7 = (3x^2 - 2x - 1)(2x + 3) + 8x - 4$$

EXAMPLE Divide $x^2 - 7x + 6$ by $x - 1$

DIVIDEND DIVISOR

$$\begin{array}{r|l} & x - 6 \quad \text{QUOTIENT} \\ \hline x - 1 \quad \text{DIVISOR} & x^2 - 7x + 6 \quad \text{DIVIDEND} \\ & x^2 - x \\ \hline & \quad -6x + 6 \\ & -6x + 6 \\ \hline & \quad 0 \quad\quad 0 \quad \text{REMAINDER} \end{array}$$

OUTCOME IS: $x^2 - 7x + 6 = (x-1)(x-6)$

EXAMPLE Divide $x^3 - 7x + 6$ by $x - 1$

DIVIDEND DIVISOR

$$
\begin{array}{r|l}
 & x^2 + x - 6 \quad \text{QUOTIENT} \\
\hline
x - 1 \quad \text{DIVISOR} & x^3 - 7x + 6 \quad \text{DIVIDEND} \\
 & x^3 - x^2 \\
\hline
 & \quad x^2 - 7x + 6 \\
 & \quad x^2 - x \\
\hline
 & \quad\quad -6x + 6 \\
 & \quad\quad -6x + 6 \\
\hline
 & \quad\quad\quad 0 \quad\quad \text{REMAINDER}
\end{array}
$$

OUTCOME: $x^3 - 7x + 6 = (x^2 + x - 6)(x - 1)$